## Profile calculation

This chapter will cover the most common calculations that serve to define the hob tooth profile.
These formulae are becoming more and more "historical" in the sense that nobody calculates by hand any more. All of these calculations have now been programmed in specific computer calculation software.
However, if you are interested in understanding the methods and concepts behind these calculations more deeply, the formulae given in this article may be of use.

_Normal profile_
The normal profile is a profile which does not undergo any particular modifications such as protuberance, semi topping or other corrections in form.
We must also consider that the rack of reference has its pitch line tangent to the theoretical pitch diameter of the gear and that therefore the pressure angle is the nominal angle of the gear.
It is important to underline this because later on we will examine hobs with lowered pressure angles which generate the desired profile even though the tooth flanks have a different inclination from the nominal inclination of the gear.
If we imagine that we must hob a gear with:

- *module* $m$
- *pressure angle* $\alpha_0$
- *circular thickness* $S_0$
- *dedendum* $h_f$
- *addendum* $h_0$
- *pitch diameter* $d_0$

the hob must have the addendum:

$$h_{kw} = h_f$$

The tooth thickness on the pitch line $S_w$ must be equal to the tooth circular thickness which in turn is equal to the pitch minus the tooth circular thickness.

$$S_{0v} = \frac{\Pi \cdot d_0}{Z} - S_0 = S_w$$

The inclination of the hob tooth flanks is equal to the pressure angle $\alpha_0$.
If the gear is not topping or semi-topping, the total depth $H_w$ must be greater than the total gear tooth depth.

$$H_w > h_k + h_f$$

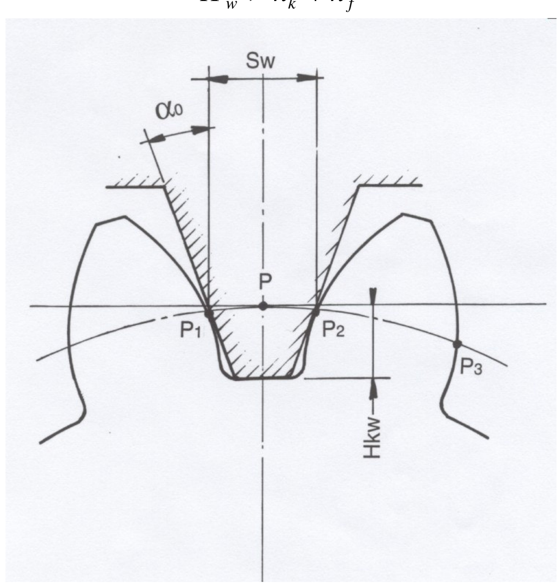

*Figure No.1*

The fact that the hob tooth profile depends on the rake angle, which is also known as the cutting angle, has been previously mentioned.
It is the angle that the cutting face forms with the plane on which the hob axis lies (in the case of grooves being parallel to the axis).
We have also already seen that in the vast majority of cases this angle is zero degrees. That means that the hob axis lies on the resharpening plane. The fundamental reason for this choice is that only in this way can the hob profile remain constant after resharpening.
Another important reason is that to resharpen hobs which have a rake angle other than zero degrees, it is necessary to position the grinding wheel out of axis by a distance that is not always the same but depends upon the real hob diameter as per the following formula:

$$tg\gamma = \frac{a}{D/2} = \frac{2a}{D}$$

In the event of a positive $\gamma$ the grinding wheel will be positioned under centre whereas in the event of a negative $\gamma$ the grinding wheel will machine above centre. See figure No.2.

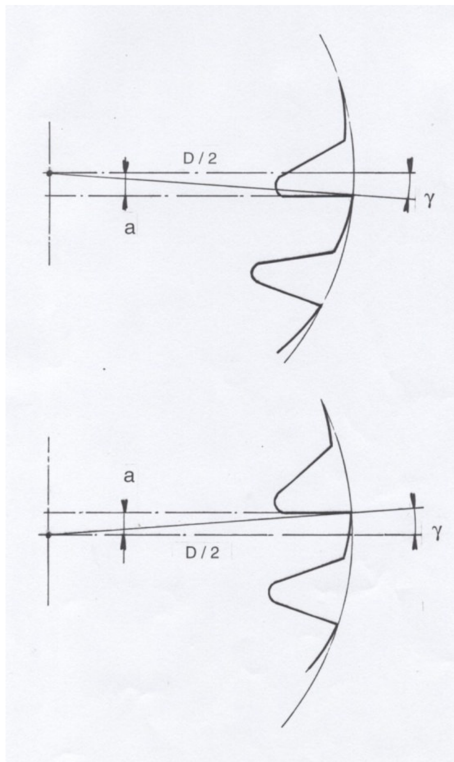


*Figure No.2*

The value **a** therefore has to be calculated each time according to the real hob diameter.
A positive rake angle is used when it is necessary to cut particularly soft material such as aluminium or very malleable steels and when the hobbing machines used cannot run at high cutting speeds.
With a positive rake angle $\gamma$ the strain on the cutting edge is reduced and its cutting action improves but at the same time the cutting edge becomes weaker as the cutting force which is always perpendicular to the cutting face surface is in an unfortunate direction in that it tends to chip the cutting edge itself.
The fact that the cutting force decreases also causes crater-type wear on the resharpening face of the tooth to form more slowly but unfortunately it is closer to the cutting edge and so increases the possibility of premature chipping.
The dynamics of chip formation is comparable to those that we would have on a tool with a single tooth. See figure No.3.

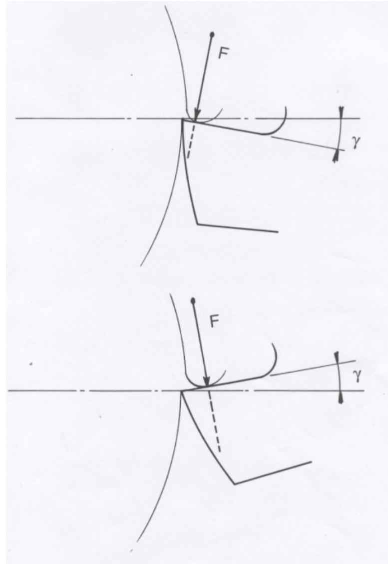


*Figure No.3*

The opposite applies to a negative angle $\gamma$. In this case the tooth becomes stronger, crater-type wear forms more quickly but further back and the cutting edges are to a certain extent protected.
A large negative rake angle is usually used for skiving-type carbide hobs which are used to finish gears that have already been heat treated.
A hob produced with a rake angle $\gamma$ other than zero must be designed and built with a pressure angle which is not equal to the normal pressure angle of the gear.
The formula that is used to calculate the hob pressure angle is the following:

$$tg\alpha_{on1} = tg\alpha_{on} \pm tg\gamma \cdot tgx$$

Where:
$\alpha_{on1}$ = *hob pressure angle*
$\alpha on$ = *normal pressure angle of gear*
$\gamma$ = *rake angle*
x = side relief angle

When producing a new hob, the plus sign is considered for positive rake angles and the minus sign for negative rake angles.

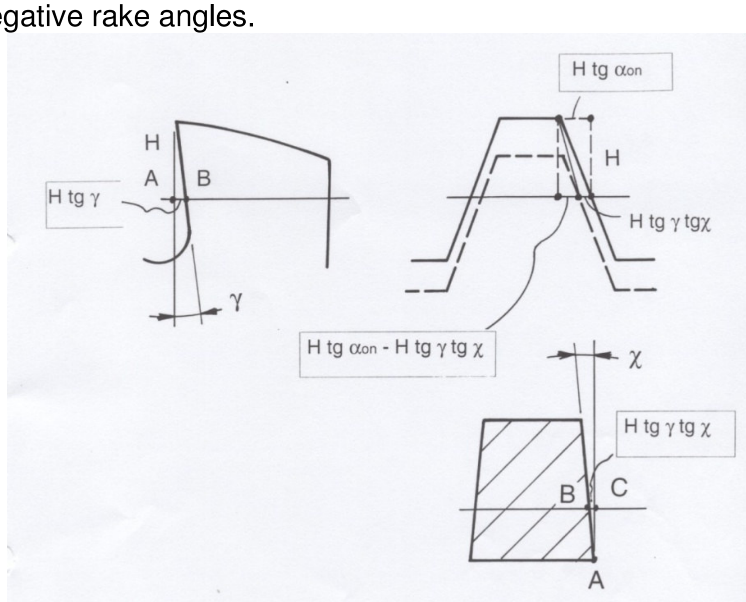


*Figure No.4*

*The interference problem*

When two gears mesh, there may sometimes be a problem of interference as the tooth tip edge of one gear may interfere with the tooth base of the other, making its revolution impossible or precarious.

Not even special cases like the meshing of a gear and a rack – a gear with an infinite number of teeth - are immune to this phenomenon.

If we therefore consider the teeth of a hob to be a rack, the interference problem may occur in certain circumstances, resulting in an undercut at the tooth base. (see figure No.5).

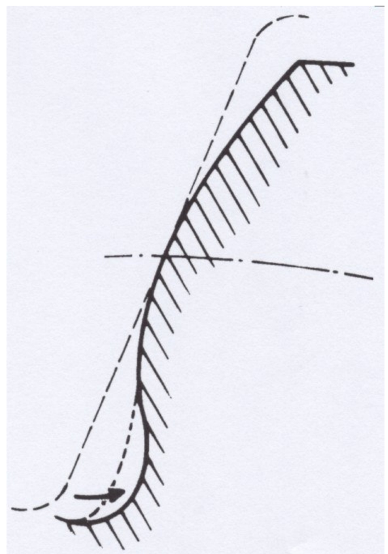

*Figure No.5*

To a certain extent this fact may be considered positive as regards tooth geometry as the undercut is like a kind of widening which is normally achieved by protuberance and it makes the finishing operation easier.

If the undercut is too big, however, the active profile is reduced to the point where the meshing line between the gears is also considerably reduced.

Nowadays gearboxes are smaller than in the past although they transmit relatively high levels of power. Long meshing lines with the longest possible active profile are therefore required.

The start of the active profile (SAP) is on a diameter which is very close to the root diameter and therefore undercuts generated by interference are not tolerable.

Another serious problem is the fact that in the zone of interference, more material is removed by each tooth, the chip thickness increases and more strain is generally put on each single hob tooth.

The phenomenon of interference between two gears depends on the pressure angle, on the transmission ratio and on the addendum to be applied on the gears.

If we consider normal dimensions where the gear addendum is equal to the module, i.e. $h_0 = k \cdot m$ , with k=1 and for hobs where $Z_1$ is infinite, i.e. where: $\frac{Z_2}{Z_1} = 0$, it is possible to use the diagram in Figure No.6 to identify the minimum number of teeth which generate interference that would enable the hob to be designed without protuberance.

*Hobs with lowered pressure angles*

First of all a few introductory remarks are required.

On the hob in particular the module and the pressure angle are conventional values which may be adjusted within ample limits without substantially influencing the workpiece produced.

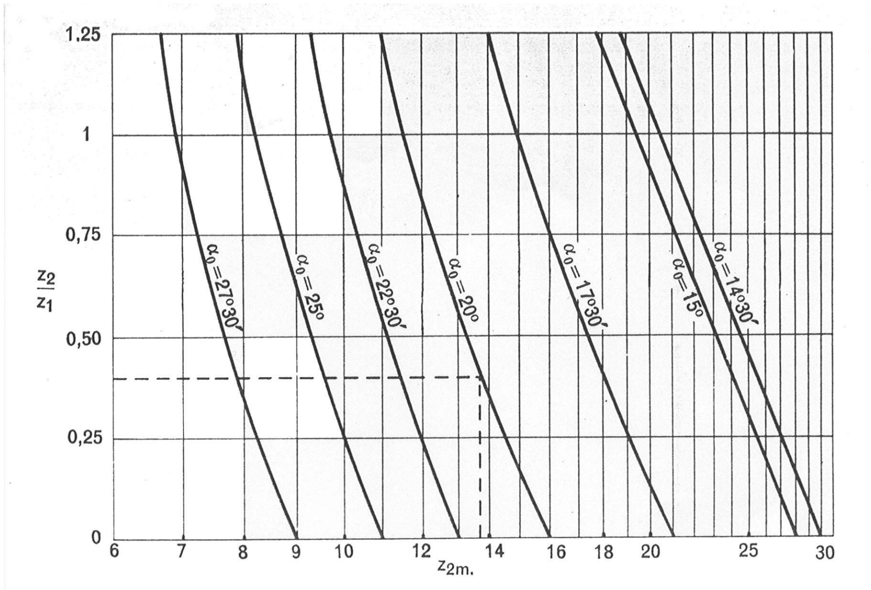


*Figure No.6*

On the hob in particular the module and the pressure angle are conventional values which may be adjusted within ample limits without substantially influencing the workpiece produced.

Normally the pitch diameter of the gear is chosen as the rolling circle. The operating pitch diameter is therefore made to coincide with the nominal pitch diameter of the gear but this is not obligatory. In theory it is possible to choose any diameter on the gear as the operating pitch diameter that is as a circle on which to roll

It is sufficient to consider the values that derive from this choice, namely the pitch value and consequently the module, the new pressure angle and the new helix angle.

Figure No.7 shows both a standard gear and next to it the same gear where a smaller pitch diameter and then a larger pitch diameter have been considered.

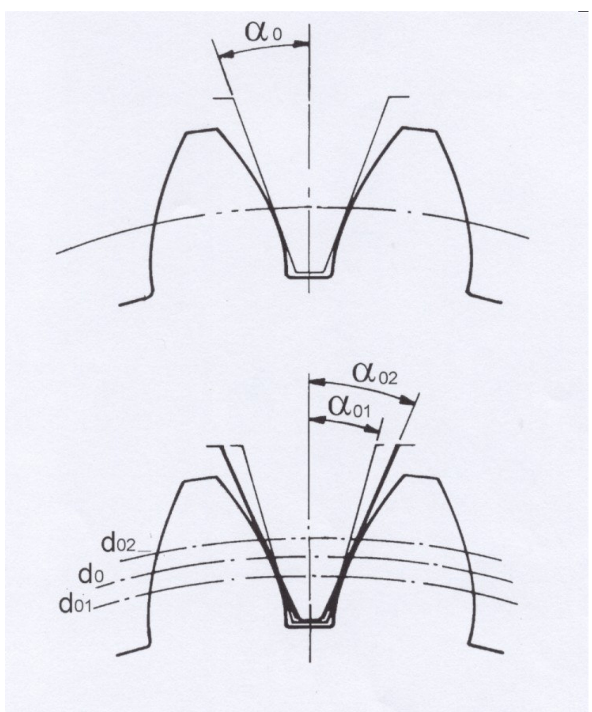


*Figure No.7*

*D0 = nominal pitch diameter (a0 = nominal pressure angle)*

*D01 = lowered pitch diameter (a01 = lowered pressure angle)*

*D02 = increased pitch diameter (a02 = increased pressure angle)*

Let us take the example of a straight-toothed hob with the following nominal data:
m = 2 mm ; Z = 30 ; $\alpha_0$ = 20°.
the pitch diameter will be:

$$d_0 = m \cdot Z = 2 \cdot .30 = 60 \text{ mm}$$

In correspondence with this diameter, the pressure angle will be 20°.
The base diameter of this gear will be:

$$d_b = d_0 \cdot \cos\alpha_0 = 60 \cdot 0{,}9397 = 56{,}382 \quad \text{mm}$$

Let us then consider two cases, choosing a gear pitch diameter of first 59 and then 61 mm. In the Table No.2 the fundamental elements which regulate these choices are shown although the gear is the same. We could also consider the new values as the nominal ones!

*Table No.1– Example of a pressure angle adjustment*

| | Nominal value | First option | Second option |
|---|---|---|---|
| $m = \frac{p}{\Pi}$ (mm) | 2 | 1,9667 | 2,0333 |
| Z | 30 | 30 | 30 |
| $\alpha_0$ | 20° | 17,1376° | 22,4370° |
| $d_0$ (mm) | 60 | 59 | 61 |
| $d_b = d_0 \cdot \cos\alpha_0$ | 56,382 | 56,382 | 56,382 |
| $\cos\alpha_0 = \frac{d_b}{d_0}$ | 0,9397 | 0,9556 | 0,9243 |
| $p = \frac{\Pi \cdot d_0}{Z}$ | 6,2832 | 6,1785 | 6,3879 |

Having said this and considering the interference phenomenon as mentioned in the previous point, this is as great as the hob addendum that is it is as large as the distance from the rolling circle to the tooth tip. Therefore to reduce interference, it is sufficient to reduce the hob addendum.

Basically it is necessary to roll on a circle that is nearer the root diameter of the gear.
If there were interference between the hob and the workpiece in the previous example, it would be lessened if option 1 were chosen while it would increase if option 2 were applied.
In this case the hob should have a pressure angle of 17,1376° , that is lower than the nominal gear pressure angle.
This is therefore the reason why we refer to these hobs as hobs with lowered pressure angles.
The biggest advantage that is obtained from using these hobs, apart from reducing the undercut and increasing the length of the active profile, is that less strain is put on the tooth tip. The performance of the hob therefore improves.
Another advantage is that the tooth base radius is also reduced which facilitates the design and use of shaving cutters.
A hob with a lowered pressure angle also tends to transmit manufacturing and assembly errors to the profile of the hobbed tooth to a lesser extent.
It is also possible to make the opposite correction as in some gears the pitch diameter is too close to, or even less than, the root diameter of the gear. In such cases it is better to design the hob with an increased pressure angle, thereby also increasing the rolling circle diameter.

*Calculation of the semitopping profile*

So far only mathematical formulae that contain trigonometric functions have been used but to calculate the semitopping profile it is necessary to introduce a new function which is often used in the field of gears and namely $inv\alpha$. To do this, the involute circle has been defined below which, as all engineers know, is the usual form of cylindrical gear teeth used in transmissions.

Definition of involute. The involute to a circle is the locus of points from the extremity of a half line which rolls on the radius circle $R_b$ known as the base circle.
See figure No.8.
Basically it is as if the base circle were wrapped up in cotton. If this cotton were unwound and held tightly, the extremity of the cotton would describe the curve formed by the involute to a circle.
All teeth on cylindrical gears have a profile which is part of the involute to a circle.

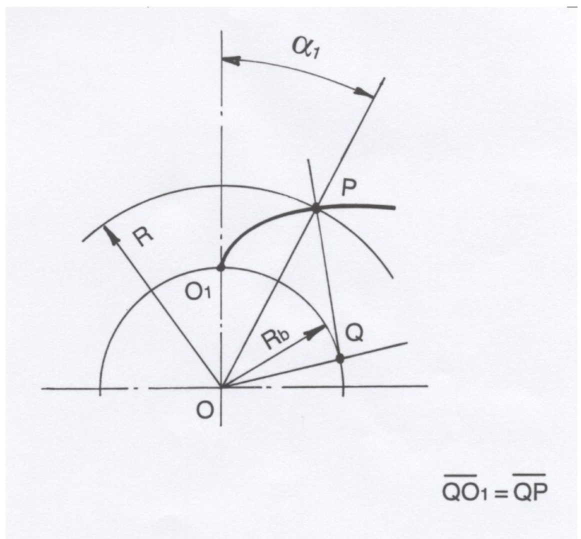


*Figure No. 8*

This curve has some interesting properties.
a)- The PQ segment has the same length as the circle arc $O_1Q$
b)- All PQ type straight lines which link a general point P of the involute to a circle are tangent to the circle itself.
c)- The same straight lines are always perpendicular to the involute.
Furthermore with reference to figure No.9 the following formulae apply.

$$tg\alpha = \frac{PQ}{OQ} = \frac{PQ}{R_b}$$

arc $(\alpha_1 + \alpha) = \frac{QO_1}{R_b} = \frac{PQ}{R_b}$ (angle expressed in radians)

$\alpha_1 = tg\alpha - \alpha$ This is the involute function of the angle $\alpha$.

$\alpha$ is known as the involute pressure angle on point P, that is it refers to radius R. This important relation is: $R_b = R\cos\alpha$ .
In practically all books on gears and in many gear cutting tool manuals, the function tables of $inv\alpha = \alpha - tg\alpha$ are published.

Nearly always gears have chamfers on the tooth tips which protect the cutting edges from dents and burrs which would make the gear noisy.
The hobs which make the tooth tip chamfers are known as semitopping hobs.
The chamfer is obtained by opportunely shaping the base of the hob teeth.

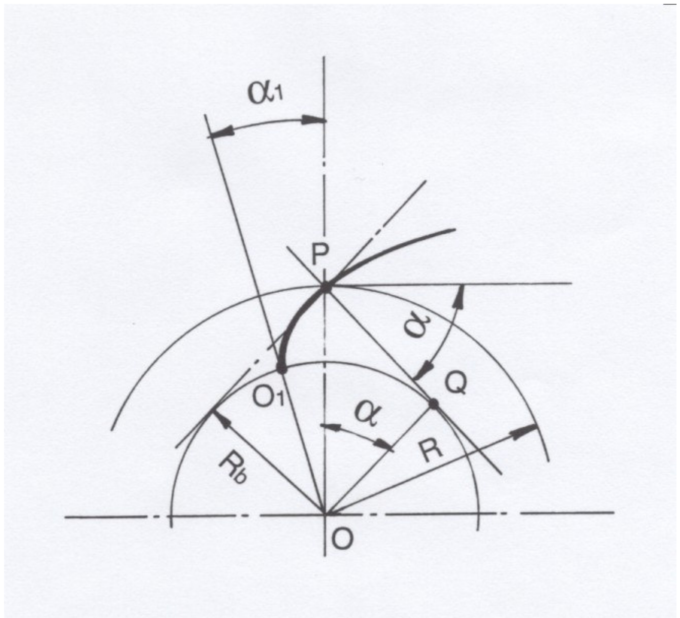


*Figure No. 9*

The two values that must be calculated for this are: the inclination $\gamma_{px}$ of the part of the hob profile that actually makes the chamfer and the distance K from the start of the chamfer from the pitch line. See figure No.10 and figure No.11.

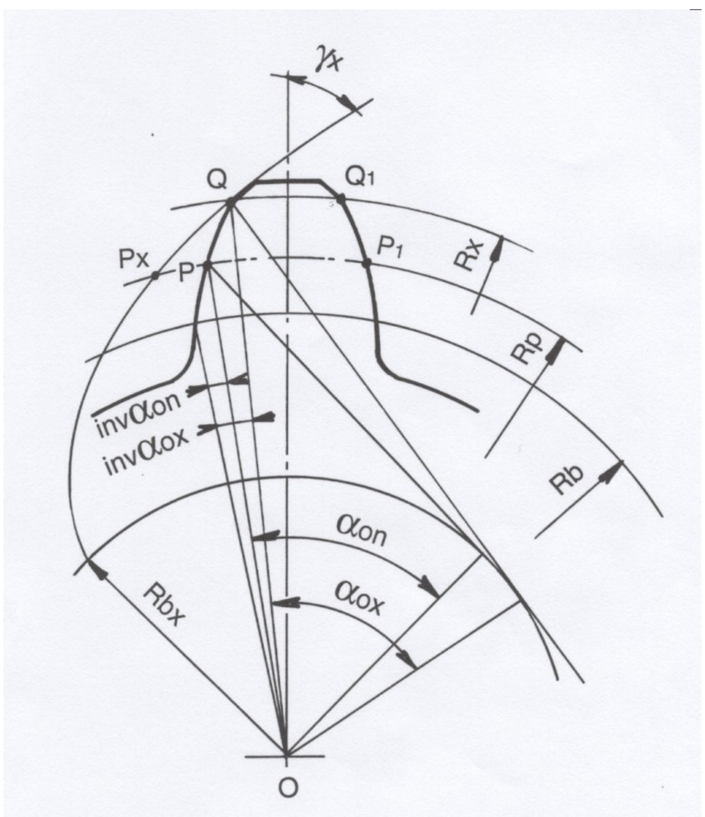


*Figure No. 10*

Basically, having a certain inclination $\gamma_x$ and starting on radius $R_x$ , the gear chamfer is a part of the involute which has a certain base radius $R_{bx}$ .

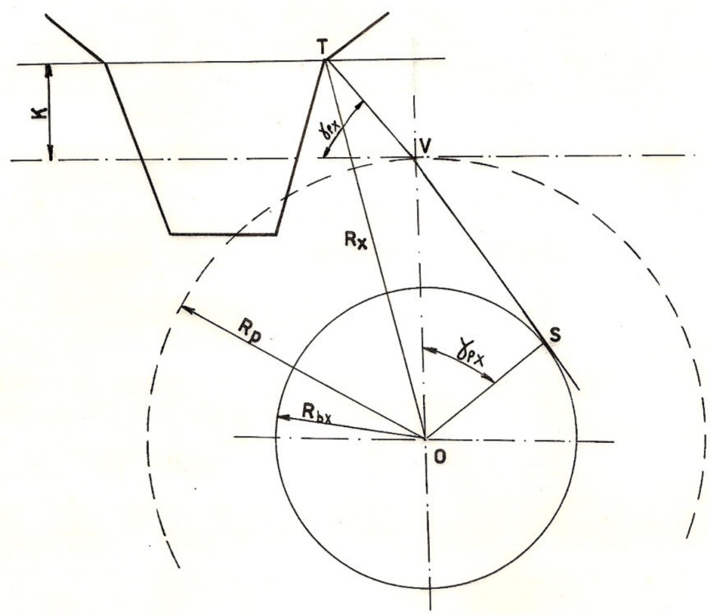


*Figure No.11*

It is important to calculate the tooth circular thickness on the radius where the chamfer begins Rx, i.e. the arc between points Q and $Q_1$. The following formula calculates this:

$$QQ_1 = \left[\frac{S_c}{2R_p} - (inv\alpha_{ox} - inv\alpha_{on})\right]\cdot 2R_x = S_c\frac{R_x}{R_p} - 2R_x(inv\alpha_{ox} - inv\alpha_{on})$$

The hob always has the same pitch line, tangent to the pitch circle which has a radius $R_p$ . The part which performs the chamfer is as if it had to cut a gear with a pitch radius equal to $R_p$ which must produce a pressure angle $\gamma_{px}$ on point Q (on radius $R_x$). Therefore:

$R_{bx} = R_x \cdot \cos\gamma_{px}$ Base radius of the involute (the chamfer is part of this involute)

$\cos\gamma_{px} = \frac{R_{bx}}{Rp}$ where $\gamma_{px}$ is the pressure angle on the pitch diameter i.e. on point $P_x$ .

This is therefore the inclination of the part of the hob profile which must produce the chamfer.
The distance K can also be calculated.

$ST = \sqrt{R_x^2 - R_{bx}^2}$ $\qquad VS = R_p \cdot \text{sen}\,\gamma_{px}$

$$TV = ST - VS = \sqrt{R_x^2 - R_{bx}^2} - R_p \cdot \text{sen}\,\gamma_{px}$$

$$K = TV \cdot \text{sen}\,\gamma_{px} = \text{sen}\,\gamma_{px}\left[\sqrt{R_x^2 - R_{bx}^2} - R_p \cdot \text{sen}\,\gamma_{px}\right]$$

The size of the chamfer on the gear always remains constant after each hob resharpening because, as previously mentioned, the hob is substantially a constant form profile milling cutter by virtue of the particular relief curve of the tooth.
However there are some considerations to be made as far as the various tolerances are concerned as these can make the effective constancy of the chamfer difficult to maintain.
Firstly there is the manufacturing tolerance of the hob itself both as regards the tooth thickness and the value of the start of the chamfer K as well as the angle $\gamma_{px}$.

There is then the tolerance of the cordal thickness of the gear teeth during hobbing which allows the hob to move more or less closely to the workpiece. Lastly there is the tolerance on the outside diameter of the gear.
It is clear that if the radius of the start of the chamfer $R_x$ remains constant, when the outside radius $R_e$ varies, the size of the chamfer may vary significantly.
Sometimes this may even cause the tip thickness, that is the thickness of the tooth on the outside diameter, to become practically null. In this case the two chamfers, right and left, intersect the outside diameter. This is never tolerated.
In some cases, especially on small module hobs (e.g. m=1 or m=1,25), the so-called topping hob is used.
In topping hobs the hob tooth profile is built in such a way that not only does the hob create chamfers but it also generates the outside diameter. This means that there is always a flat plane at the tip of the tooth.

*Verifying the value of semitopping*

Semitopping hobs are designed to obtain a chamfer with a determined radial entity and with a given inclination on gears that have a fixed number of teeth.
Therefore a hob of this kind will produce a chamfer exactly as planned and only on the gear for which it has been calculated.
If you want to cut another gear which has the same characteristics but a different number of teeth, the value of the chamfer and its inclination will vary. This variation may be tolerated only within certain limits.
The value of the chamfer increases if the number of teeth of the gear to be hobbed increases compared to the number of teeth of the gear for which the hob was originally designed and vice versa.
Once the limits within which the entity of the chamfer may be tolerated have been determined, the following verification is then carried out to see if the hob will produce the chamfer within these limits.
Considering the values $K, \alpha_{0n}, \gamma_{px}, m$ of the hob and the number of teeth Z of gear teeth on which the chamfer is to be produced, the following apply:

$R_p = \frac{m \cdot Z}{2}$ gear pitch diameter

$R_b = R_p \cdot \cos\alpha_{0n}$ radius of base circle

$R_{b'} = R_p \cdot \cos\gamma_{px}$ radius of base circle of chamfer

It is then possible to calculate:

$$A = \frac{K \cdot sen(\gamma_{px} - \alpha_{on})}{R_p \cdot \cos\gamma_{px} \cdot \cos\alpha_{on}} + (inv\gamma_{px} - inv\alpha_{on})$$

Once the maximum and minimum chamfer radial values have been determined $C_{max}$ and $C_{min}$ the respective diameters of the start of the chamfer are found.

$R_{x1} = R_e - C_{max}$ and $R_{x2} = R_e - C_{min}$

It is therefore possible to calculate the chamfer and involute pressure angles in correspondence with these diameters.

$\cos\gamma_{px1} = \frac{R_{b'}}{R_{x1}}$ $\cos\alpha_{x1} = \frac{R_b}{R_{x1}}$

$$\cos\gamma_{px2} = \frac{R_{b'}}{R_{x2}} \qquad \cos\alpha_{x2} = \frac{R_b}{R_{x2}}$$

Lastly the value of parameter A' is calculated:

$$A_1' = inv\gamma_{px1} - inv\alpha_{x1} \qquad A_2' = inv\gamma_{px2} - inv\alpha_{x1}$$

and therefore:

for $C_{max}$ it must be $A_1' \leq A$
for $C_{min}$ it must be $A_2' \geq A$

If these relations are applied the hob may be used as it will produce the chamfer within the required limits.

*Protuberance and full radius*
A profile with protuberance is applied when it is necessary to widen the gear tooth base to facilitate subsequent shaving or grinding operations.
As mentioned before, in some conditions this widening may be generated naturally thanks to the phenomenon of interference, especially on gears with a low number of teeth.
It is important to keep this in mind as protuberance should only be used when it is really necessary.
In fact hobs with protuberance tend to wear more quickly and therefore, in general, their performance is inferior compared to hobs without protuberance.
In figure No. 12 a tooth with protuberance is illustrated. We can observe that the part **c** which joins the area of the normal profile with the external part of the protuberance, has a lower inclination than the pressure angle and in this area the side relief is lower.
Therefore in this zone there will be a tendency for greater wear.

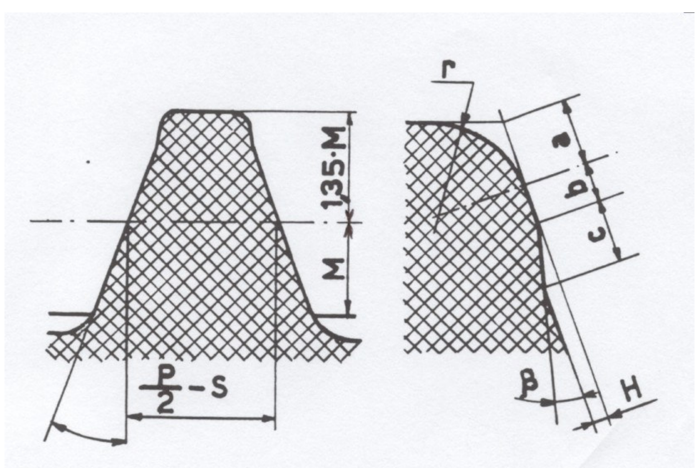


*Figure No.12*

For $\alpha = 20°$ ; $\beta = 5°$
For $\alpha = 14°30' - 15°$ ; $\beta = 3°30'$

However, when hobbing gears with a high number of teeth which must then be shaved or ground or in any case finished by another process, it is advisable to use hobs with protuberance to avoid excess strain near the tooth base radius to avoid the formation of small “steps” where the shaved or ground surface ends. See figure No.30.

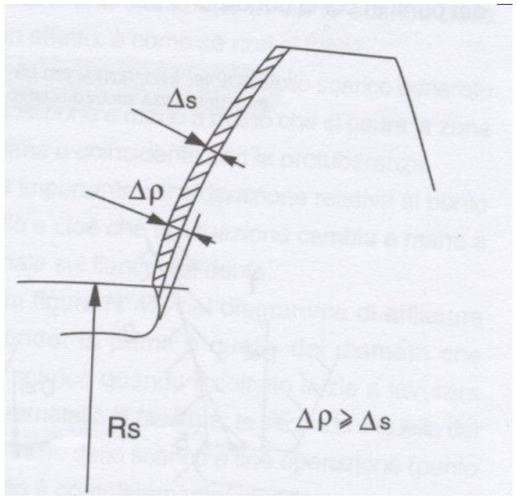


*Figure No.13*

With reference to figure No.12, in table No.2 the dimensions of protuberance for the various modules and pressure angles are shown in the event that the finishing operation is performed by shaving.
If finishing is performed by grinding, the value of protuberance depends on the stock allowance envisaged.

*Table No.2*

<table>
<tr><th rowspan="2">Module (mm)</th><th rowspan="2">Ps stock (micron)</th><th rowspan="2">H (micron)</th><th rowspan="2">r (mm)</th><th rowspan="2">b (mm)</th><th colspan="2">For a press. Angle of 20°</th><th colspan="2">For a press angle of. 14°30' - 15°</th></tr>
<tr><th>a (mm)</th><th>c (mm)</th><th>a (mm)</th><th>c (mm)</th></tr>
<tr><td>1</td><td>15 - 40</td><td rowspan="4">25 - 40</td><td rowspan="2">0,40</td><td rowspan="2">0,13 - 0,20</td><td rowspan="2">0,30</td><td rowspan="4">0,35</td><td rowspan="2">0,30</td><td rowspan="4">0,50</td></tr>
<tr><td>1,25</td><td>20 - 45</td></tr>
<tr><td>1,5</td><td>25 - 50</td><td rowspan="2">0,50</td><td>0,15 - 0,24</td><td rowspan="2">0,35</td><td rowspan="2">0,40</td></tr>
<tr><td>1,75</td><td>30 - 55</td><td>0,18 - 0,28</td></tr>
<tr><td>2</td><td>35 - 60</td><td rowspan="4">40 - 50</td><td>0,65</td><td>0,20 - 0,30</td><td>0,45</td><td rowspan="4">0,50</td><td>0,50</td><td rowspan="5">0,75</td></tr>
<tr><td>2,25</td><td>40 - 65</td><td rowspan="2">0,75</td><td rowspan="2">0,25 - 0,39</td><td rowspan="2">0,55</td><td rowspan="2">0,60</td></tr>
<tr><td>2,5</td><td rowspan="3">45 - 70</td></tr>
<tr><td>2,75</td><td rowspan="3">0,90</td><td rowspan="2">0,30 - 0,45</td><td rowspan="3">0,65</td><td rowspan="3">0,70</td></tr>
<tr><td>3</td><td rowspan="8">50 - 65</td><td rowspan="7">0,65</td></tr>
<tr><td>3,25</td><td rowspan="2">50 - 75</td><td rowspan="3">0,35 - 0,55</td><td rowspan="8">0,95</td></tr>
<tr><td>3,5</td><td rowspan="2">1,00</td><td>0,70</td><td rowspan="2">0,80</td></tr>
<tr><td>3,75</td><td>55 - 80</td><td>0,75</td></tr>
<tr><td>4</td><td>60 - 85</td><td rowspan="2">1,10</td><td>0,40 - 0,62</td><td rowspan="2">0,80</td><td>0,90</td></tr>
<tr><td>4,5</td><td rowspan="2">65 - 90</td><td>0,45 - 0,70</td><td>1,00</td></tr>
<tr><td>5</td><td>1,40</td><td>0,50 - 0,80</td><td>1,00</td><td>1,10</td></tr>
<tr><td>5,5</td><td>75 - 100</td><td>1,60</td><td>0,55 - 0,85</td><td>1,20</td><td rowspan="2">0,70</td><td>1,20</td></tr>
<tr><td>6</td><td rowspan="2">80 - 105</td><td rowspan="4">65 - 75</td><td>1,70</td><td>0,60 - 0,95</td><td>1,30</td><td>1,40</td></tr>
<tr><td>6,5</td><td>1,90</td><td>0,65 - 1,00</td><td>1,40</td><td rowspan="3">0,80</td><td>1,50</td><td rowspan="3">1,10</td></tr>
<tr><td>7</td><td>85 - 110</td><td>2,20</td><td>0,75 - 1,10</td><td>1,60</td><td>1,70</td></tr>
<tr><td>8</td><td>90 - 115</td><td>2,50</td><td>0,85 - 1,25</td><td>1,80</td><td>1,95</td></tr>
</table>

Full radius hobs have a profile like that indicated in figure No.14.

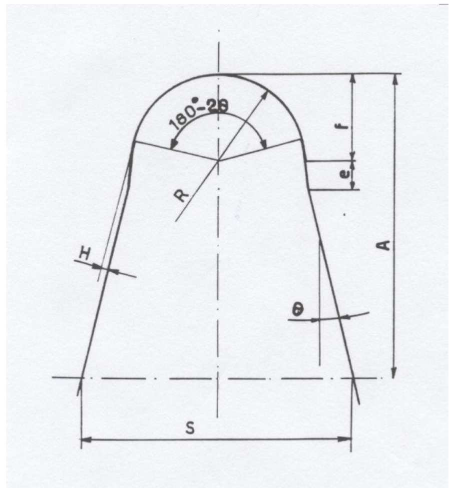


*Figure No.14*

Normally this full radius on the tooth tip is associated with protuberance and its primary purpose is to make the reduction of the flank inclination in the protuberance area less accentuated. This results in a consequent improvement in wear resistance during use.
Secondly the full radius on the tooth tip eliminates cutting edges with small radii that are normally areas prone to chipping and worse levels of wear.
This correction is not always possible as the root diameter of the gear must be lowered significantly.